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Fiscal consolidation: an optimal control approach

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This *Focus* presents a simple model of public finance dynamics that includes the main ingredients used for policy discussions. The model distinguishes between the conjunctural and the structural deficit and assumes that the latter evolves slowly, as a result of budgetary adjustments that are decided by the government. These budgetary adjustments have a multiplier effect on output, governed by the parameter m. Hence, the model captures, in three simple equations, the essence of public finance dynamics as used by macroeconomic forecasters and analysts.

The *Focus* then turns to a problem of optimal fiscal consolidation. Starting from an initial primary deficit, a planner wants, on the one hand, to smooth the adverse effects on the output gap over time, and on the other hand, to avoid an excessive rise in public debt. When the real interest rate r equals the rate of potential growth g*, the optimal control solution to this problem shows that optimal fiscal consolidation is frontloaded, with a budgetary adjustment that is linear in time, and a speed that depends on the social cost of debt λ/m^2 relative to the multiplier.

The *Focus* then discusses practical issues for countries, such as France, for which $r = g^*$ is a good approximation. Given a tolerable maximal level of debt/GDP at the end of consolidation, as well as a maximal initial budgetary adjustment, it is possible to obtain bounds on the optimal stopping time T, and therefore the implicit weight λ/m^2 placed by the planner on public debt relative to the multiplier.

Applied to the French public finance situation in 2024, with a structural primary deficit of 3.2 percent of GDP and a public debt level of 110 percent of GDP, if the maximal tolerable budgetary adjustment in the first year is 1 percent of GDP and the maximal level of debt is 120 points of GDP, we obtain an optimal consolidation horizon of $T = 7$ years.

Model

Here we present a canonical model of public finance dynamics, similar to those used by macroeconomic analysts, which captures the essential concepts used in policy discussions. As an example, this model is taught at the French national school of administration (ENA). It is also used, in a more complex version, by the OFCE public debt observatory (DebtWatch).[1]

The model is cast in continuous time, which makes for simpler analytical solutions and allows for graphical intuition. The equation for the dynamics of the public debt-to-potential GDP ratio b_t is given by:

$$\dot{b_t} = (r_t - g_t^*)\, b_t + d_t - \epsilon OG_t \tag{1}$$

Here, $\dot{b_t}$ is the derivative of the debt-to-GDP ratio with respect to time, r_t is the instantaneous real interest rate, g_t^* the instantaneous rate of potential growth, and the primary deficit is split between a structural component d_t, controlled by the government, and a conjunctural component, which we assume to have an elasticity ϵ with respect to the output gap OG_t (meaning that an output gap of 1 point improves the deficit-to-GDP ratio by ϵ points.)

The government has initial debt of $b_0 = b$ points of GDP, and an initial structural primary deficit of $d_0 = d$ points of PIB. It picks the time path of the budgetary adjustment BA_t, which affects the time derivative of the structural primary deficit-to-GDP ratio $\dot{d_t}$:

$$\dot{d_t} = BA_t \tag{2}$$

Hence, when BA_t is negative, the structural primary deficit slowly improves relative to GDP. For instance, a negative BA corresponds to a situation where the growth rate of public spending is lower than the potential growth rate of GDP, and/or where the tax-to-GDP ratio is increased. These are the two ways in which fiscal consolidation can be done in practice.[2]

On the other hand, a budgetary adjustment has an impact on the output gap via the multiplier m, that is:

$$OG_t = m \cdot BA_t \tag{3}$$

Hence, a budgetary adjustment lowers GDP relative to potential GDP.[3] This equation assumes that monetary policy does not sufficiently accommodate the fiscal adjustment through lower interest rates, an assumption that is realistic in practice for a country inside a monetary union. Note that in practice, the multiplier m depends on the type of fiscal consolidation (i.e., spending-based or taxed-based), as well as the specific type of spending that is reduced or taxes that are increased. For instance, in a Keynesian model, m is larger when consolidation is spending based rather than when it is taxed based, though the empirical literature does not tend to find such a sharp contrast. For our purposes, m is taken as a given, but its central place in the model highlights the practical importance of its empirical evaluation.

The optimal fiscal consolidation problem

We consider a case with $r_t = g_t^*$, which is a good approximation for France and delivers especially simple and intuitive results.[4] The government seeks to stabilize the ratio of public debt to GDP, after an optimal consolidation horizon of T, at a level b_t. Equations (1)-(3) imply that we must have $d_t = OG_t = BA_t = 0$ for $t \geq T$.

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[1] See https://ofce.shinyapps.io/debtwatchr/

[2] We can write $d_t = G_t/Y_t^* - \tau_t$ where G_t, is the level of public spending, Y_t^* is potential GDP, and τ_t is the tax rate, i.e. the ratio of tax revenue to GDP. Then, $BA_t = \dot{d_t} = G_t/Y_t^*(g_{Gt} - g_t^*) - \dot{\tau_t}$, when $g_{Gt} = \dot{G_t}/G_t$ is the rate of growth of public spending and g_t^* the rate of growth of potential GDP. A budgetary adjustment of -1 point per year corresponds either to an increase in the tax rate by 1 point, or a rate of growth of public spending that is below that of potential growth by $1/(G/Y)$ points—i.e., for the case of France, about 1,8 points (since G/Y=0.55 en 2023.)

[3] The output gap $OG_t = (Y_t - Y_t^*)/Y_t^*$ is the percentage difference between GDP Y_t and potential GDP Y_t^*.

[4] The $r = g^*$ assumption is consistent with the French data on average since the 1970s. See Auclert A., T. Philippon et X. Ragot (2024) : « Quelle trajectoire pour les finances publiques françaises? », *Note du CAE*, July. In practice, there is a windfall effect with $r_t < g_t^*$ forecasted over the 2024-2030 period, which slightly lowers the increase in the public debt given a path of budgetary adjustment relative to the calculations in this *Focus*.

Hence, the problem is to find a time path for budgetary adjustments BA_t that will slowly take the primary deficit from $d_0 = d$ to $d_T = d$. We impose the constraint that $BA_t \leq 0$, that is, that the budgetary effort is sustained throughout the consolidation. The problem is to find the optimal horizon T as well as the optimal time path for BA_t for $t < T$.

We assume that the government wants to minimize public debt at the end of the consolidation b_T, but also that it wants to smooth the output gap over time, and assume that output gap has quadratic time cost.[5] Hence, the optimal consolidation problem writes:

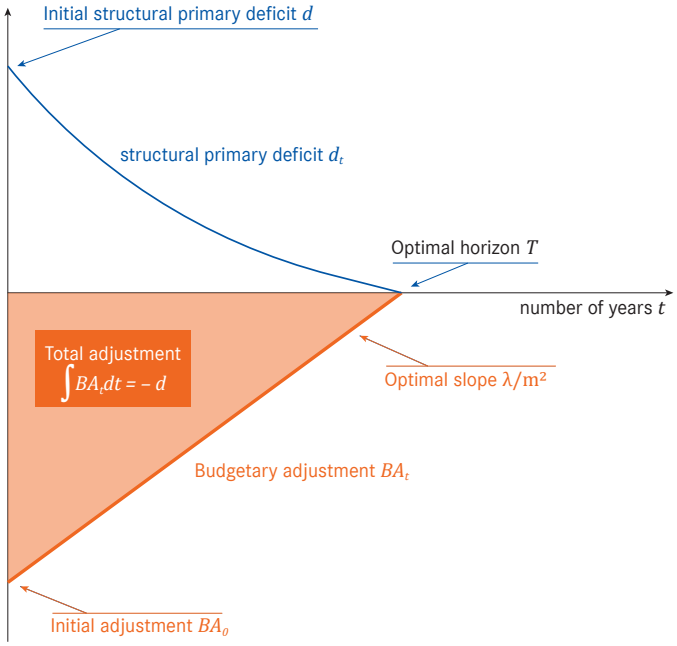
$$min \; ½ \int_0^T OG_G^2 \, dt + \lambda b_T \tag{4}$$

Here, λ stands for the social cost of an extra point of debt (for instance, due to external commitments made by the country, to the risk of a rising cost of debt, crowding-out effects, and so on) relative to that of smoothing the output gap. The government picks the path of budgetary adjustments $BA_t \leq 0$ to minimize its objective in (4) while respecting the macroeconomic constraints (1), (2), (3) as well as the border condition $\int_0^T BA_t dt = -d$.

The solution

Optimal control gives the following solution, presented in figure 1.

Figure 1. Graphical representation of the optimal fiscal consolidation solution.



At the optimum, the budgetary adjustment has the form:

$$BA_t = \frac{\lambda}{m^2}(t - T) \tag{5}$$

The intuition is that the planner wants to smooth the budgetary adjustment over time, but delaying adjustment by t years increases public debt by t points more than if the adjustment was done right away. Hence, adjustment is frontloaded, and then falls linearly in time. The border condition, which states that the structural primary deficit must reach zero at the end of the consolidation, is:

$$\frac{T \times BA_0}{2} = -d \tag{6}$$

[5] A quadratic time cost can be seen as the second order approximation to an underlying utility maximization problem. See, for instance, Woodford (2003) : « Interest and Prices: Foundations of a Theory of Monetary Policy ».

Graphically, this condition says that the area of the red triangle in figure 1 is equal to the initial structural primary deficit d.

Finally, the time path for the structural primary deficit is $d_t = d\,((t–T)/T)^2$. Integrating, this implies that public debt at the end of the consolidation is

$$b_T = b + \epsilon md + dT/3$$

Intuitively, fiscal consolidation has an incompressible effect on final debt of ϵmd, owing to the fact that consolidation has to lower the output gap and create conjunctural deficits. In addition to this, a horizon of T years implies an additional increase in debt equal to the integral under the path of the structural deficit (the blue line in figure 1), i.e. $\Delta b = dT/3$

The model has the following practical implications for the determination of the optimal horizon. First, equation (6) implies that

$$T = \frac{2d}{|BA_0|}$$

Given an initial structural primary deficit of d, a maximal budgetary adjustment of x points therefore implies a minimal horizon of $2d/x$.

In addition, we have:

$T = 3\Delta b/d$

Given a maximal tolerable increase of debt of y points (on top of the incompressible level), this delivers a maximal consolidation horizon of $3y/d$. Hence, we obtain the bound $2d/x \leq T \leq 3y/d$ for the optimal consolidation horizon. Implicitly, these calculations put a bound on the ratio λ/m^2 of the planner weight on public debt relative to the multiplier.

Application: France in 2023

The French treasury estimates that, in 2023, France had a structural primary deficit of $d = 3.2$ points of GDP. The maximal structural primary adjustment that can be done in a year is likely around $x = 1$ percent of PIB. Hence we have $T \geq 2 \times 3.2/1 = 6.4$ years. On the other hand, France starts in 2023 with a public debt level of 110 points of PIB. Given an elasticity of the conjunctural deficit to the output gap of $\epsilon = 0.57$ and a multiplier m=1 , consolidation leads to a minimal increase in public debt of $\epsilon md = 0.57 \times 1 \times 3.2 = 1.8$ points, taking debt to 111.8 points of GDP. If we think that the maximal debt level is at 120 points de PIB, we have Δb=120-111,8=8.2 points, and therefore $y = 8.2$. This implies $T \leq (3 \times 8.2)/3.1 = 7.9$ years. Hence, this calculation delivers a horizon of 7 years as optimal, corresponding to an implicit weight on public debt relative to the multiplier of $\lambda/m^2 = 2d/T^2 = (2 \times 3{,}1)/7^2 = 0.12$, to an initial budgetary adjustment of $-\lambda/m^2 \times T = 0{,}84$ points of GDP, and to a level of debt at the end of the consolidation of $b_T = b + \epsilon md + dT/3 = 110 + 1{,}8 + (3{,}1 \times 7)/3 = 119$ points of GDP.

[6] Source: Programme de Stabilité (2024) : « tableau 5: solde primaire corrigé des effets du cycle » (April). The structural deficit is estimated to be about 4.8 points of GDP, and interest costs about 1.7 points.

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