



## Simulation of a Fiscal Public Expenditure Rule Dependent on the Level of Public Debt

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### 1. Simulation of the rule for France between 1997 and 2017

In this first part, starting from the fiscal rule contained in the report “Reconciling risk sharing with market discipline: A constructive approach to euro area reform” published in January 2018 by 14 French and German economists at the Centre for Economic Policy Research, we have simulated for France between 1997 and 2017 the fiscal policy that would have resulted from the proposed rule as well as the public finances trajectory.

The report by the 14 economists indeed offers a public expenditure rule with a public debt target. The basic principle is simple: nominal public expenditures should not grow, in the long run, faster than the nominal GDP (which is the sum of the potential real GDP and expected inflation) and they should grow slower for countries that need to reduce their debt, roughly speaking countries with a debt higher than 60% of the GDP. The speed of debt reduction would depend on its distance to 60% (the further away, the faster the speed of adjustment). Public expenditures are net of interest payments, of unemployment spending (except when these are due to discretionary changes) and corrected by discretionary revenue measures (changes in tax rates and tax bases).

The proposed public expenditure rule can therefore be written as follows:

$$\frac{G_{P,t} - G_{P,t-1}}{G_{P,t-1}} = \frac{\bar{Y}_t - \bar{Y}_{t-1}}{\bar{Y}_{t-1}} + \frac{NM_t - NM_{t-1}}{G_{P,t-1}} + \dot{p}_t - \gamma \left( \frac{D_{t-1}}{Y_{t-1}} - 0,6 \right)$$

where  $G_{P,t}$  is the primary public expenditure in value excluding unemployment spending (in current euros)

NM: new discretionary revenue measures (changes in tax rates and tax bases) (in current euros)

$\bar{Y}_t$ : potential GDP in volume

$\dot{p}_t$ : expected inflation

$D_t$ : gross public debt as defined by the Maastricht criteria (in current euros)

The adjustment parameter  $\gamma$  of public expenditure according to the deviation from the public debt target is set in the following way:

$$\gamma = 0,02 \text{ if } \frac{D_{t-1}}{Y_{t-1}} > 0,9; \gamma = 0,01 \text{ if } 0,6 < \frac{D_{t-1}}{Y_{t-1}} < 0,9; \gamma = 0 \text{ if } \frac{D_{t-1}}{Y_{t-1}} < 0,6$$

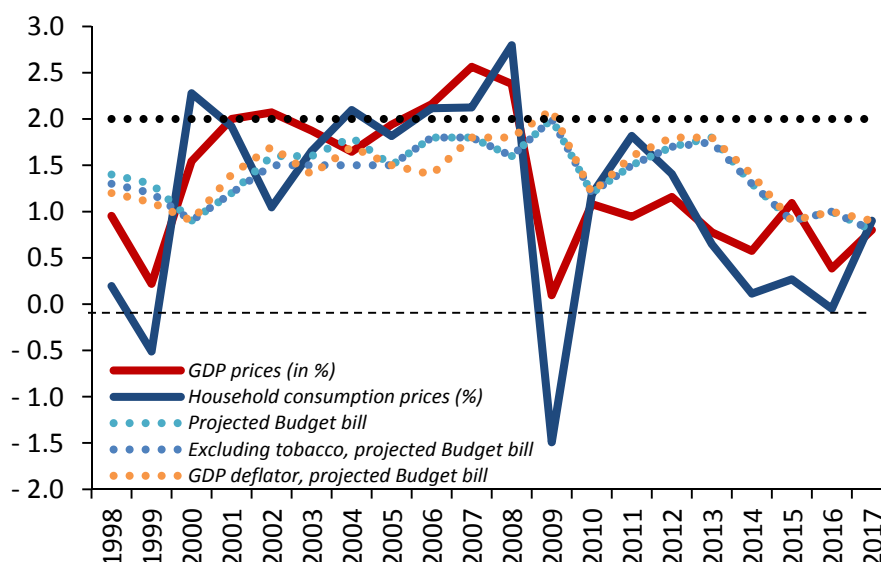
## 1.1. The public expenditure rule and the different parameters

As we may notice, this rule depends on several parameters, the first one being the restoring force of the public debt to 60% of the GDP. In this part, we do not discuss this essential parameter as it will be done in the second part. However, we focus here on the two other parameters of the rule: the potential GDP growth and expected inflation.

### 1.1.1. Expected inflation

The public expenditure rule will depend on expected inflation. It is therefore important to define what is meant by expected inflation in the expenditure rule. Should we keep the measure of consumer prices, its underlying price or the one of GDP prices. Or even retain an inflation target, regardless of the vagaries of the economy. As we may notice, inflation measured by consumer prices is more volatile than the one measured by GDP prices (Figure 1). Moreover, the sustainability of public expenditure is measured by the standard of nominal potential GDP trajectory. It would therefore be logical to use the expected growth of GDP prices to calculate the public expenditure rule, or to keep a 2% inflation target, consistent with the medium-run objectives of the ECB. Here, the expected inflation, whether it is measured by consumer prices or by GDP prices, is the one that stems from the different budget laws that were voted between 1997 and 2017<sup>(1)</sup> (Figure 1).

**Figure 1. Expected and realized inflation, in %**



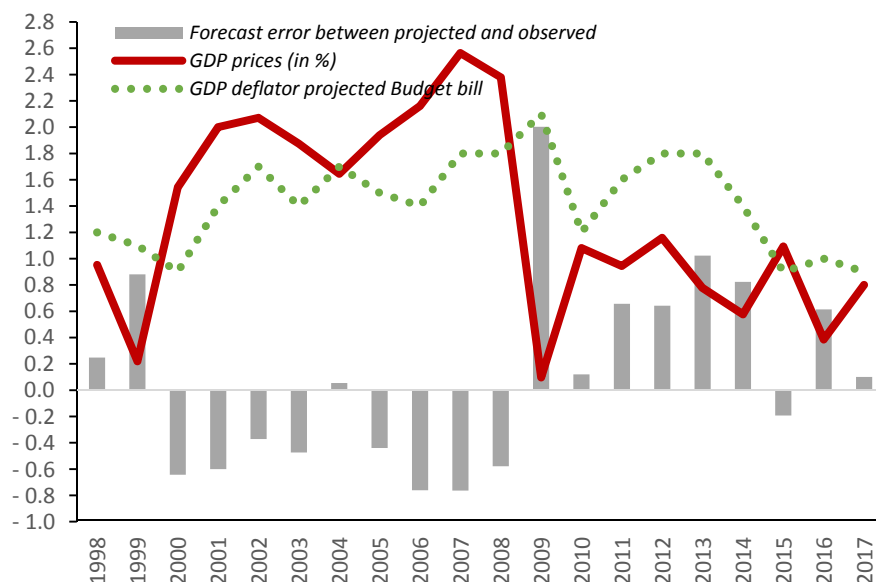
Sources: INSEE, Budget bill.

If we only use inflation calculated from GDP prices, which is the most consistent with the rule, we can notice a significant gap between the GDP deflator observed and the one projected by the various budget laws. The forecast underestimates real price variations. The forecast is usually lower than the inflation

(1) For example, the anticipated inflation for 2007 comes from the 2007 Budget Bill published at the autumn of 2006.

observed in periods of sustained growth (and vice versa): on average it was underestimated by 0.5% per year between 2000 and 2008 and overestimated by 0.6% per year between 2009 and 2017 (Figure 2). Based on past observations of the price forecasts from the budget bill, which is less cyclical than the real one, the expected inflation chosen to predict the evolution of public expenditure would increase the counter-cyclical nature of the rule. However, this analysis is based on a past observation without assurance of its reproduction in the future. Thus, a systematic prediction error with the opposite sign would, however, make the rule more pro-cyclical.

**Figure 2. Predicted and realized GDP Deflator, in %**



Sources: INSEE, Budget bill.

### 1.1.2. The potential GDP growth rate

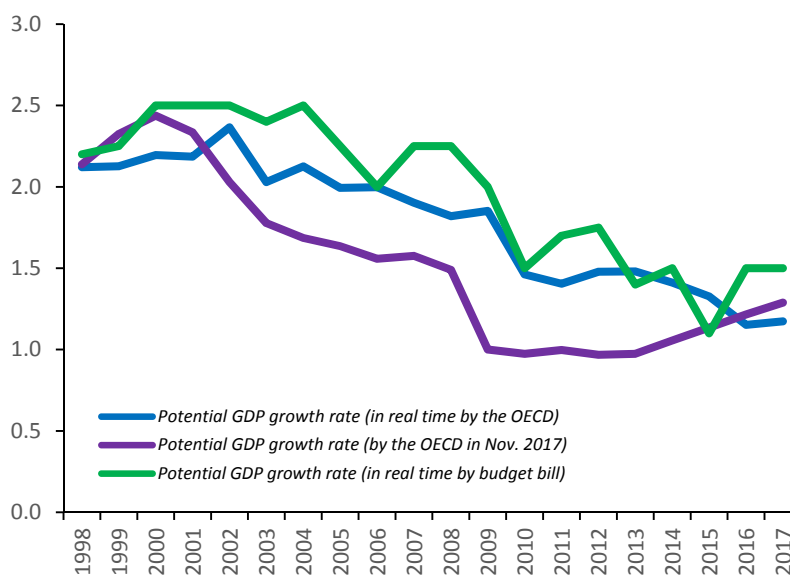
The measure of potential GDP is critical to define the public expenditure trajectory resulting from the rule. Since this variable is unobservable, with a measure resulting from different evaluation methods, it does not lead to a consensus in its assessment. And with the same method, its trajectory can be revised over the years depending on past growth and observed cycles. Also, it is important to distinguish, in the evaluation of the potential growth, its real-time measure from the one obtained ex post, namely the one obtained over the whole period with the known growth trajectory. For our analysis, we kept three measures:

- the one conducted in real-time by the OECD from each of its publication vintages
- the one conducted ex post by the OECD from the last known vintage (November 2017) as part of the exercise carried out
- the one conducted in real-time from the different Budget bills

Finally, we rejected the possibility of a constant potential GDP growth over the whole period.

As we may notice the potential growth trajectories are different according to the evaluation method chosen and vary greatly overtime, the two most favorable being those measured in real-time. The real-time one from the Budget bills is on average of 2% over the period and of 1.8% for the real-time OECD one. On the other hand, it falls to 1.5% on average over the period if we keep the OECD's current view according to its last publication (Figure 3).

**Figure 3. Potential and actual GDP growth rate, in %, in volume**



Sources: INSEE, OECD, Budget bills.

## 1.2. Simulation of the rule for the 1997-2017 period

The simulation of the public expenditure rule requires taking into account the discretionary revenue measures (changes in tax rates and tax bases) over the period of interest. Consequently, we have measured the impact of the discretionary measures on tax revenues for each year, from the evaluation in the Economic, Social and financial reports included in the different Budget bills. This allows to filter in the tax revenue dynamics the effects of the tax bases.

Our analysis for the rule applies each year independently from the others, the public debt here being considered as exogenous for the period.

### 1.2.1. Impact on the expenditure growth rate and fiscal impulse

In order to perform the public expenditure simulations between 1997 and 2017, we have selected five types of assumptions:

- Two expected price assumptions:
  - o The growth rate of GDP prices forecast from each Budget bills;
  - o A fixed inflation target of 2% for the whole period;
- Three potential growth assumptions:
  - o The potential growth estimated by the OECD for the period 1997-2007 in its November 2017 publication;
  - o The real-time potential growth forecast by the OECD in its different vintages of publication (for example: the potential growth forecast for 2007 from the autumn 2006 OECD publication);
  - o The real-time potential growth forecast in each Budget bills (for example: the potential GDP growth rate forecast for 2007 from the 2007 Budget bill published at the autumn 2006).

Between 1997 and 2017, the primary public expenditure, in current euros, has risen by 3.3% on average each year (Figure 4).

The spending rule would be more restrictive than what was done between 1997 and 2017 in 2 cases:

- GDP prices from the Budget bill and potential growth from the OECD's last vintage: 2.8%
- GDP prices from the Budget bill and real-time potential growth from the OECD: 3.0%

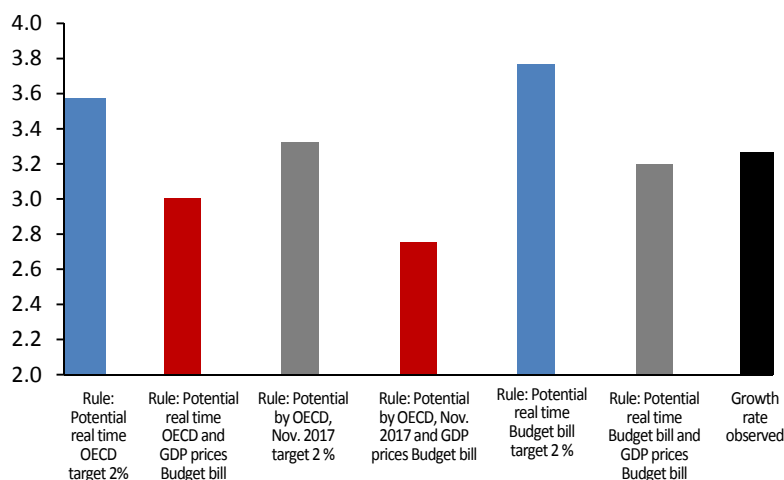
The spending rule would be more expansive than what was done between 1997 and 2017 in 2 cases:

- Inflation target at 2% and real-time potential growth from the Budget bill: 3.8%
- Inflation target at 2% and real-time potential growth from the OECD: 3.6%

The spending rule would be close to that what was done between 1997 and 2017 in two cases:

- GDP prices and potential growth from the Budget bill: 3.2%
- Inflation target at 2% and potential growth from the OECD's last vintage: 3.3%.

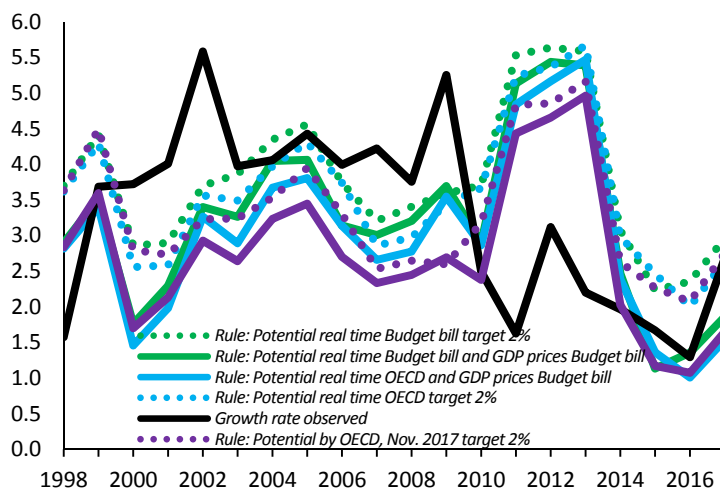
**Figure 4. Growth rate for primary public expenditure observed and predicted from the rule (annual average over the period 1997-2017), in %, in current euros**



Sources: INSEE, OECD, Budget bills, OFCE calculations.

In all the cases selected, although there are nuances according to the assumptions adopted, the rule is more counter-cyclical than the observed public expenditure trajectory (Figure 5).

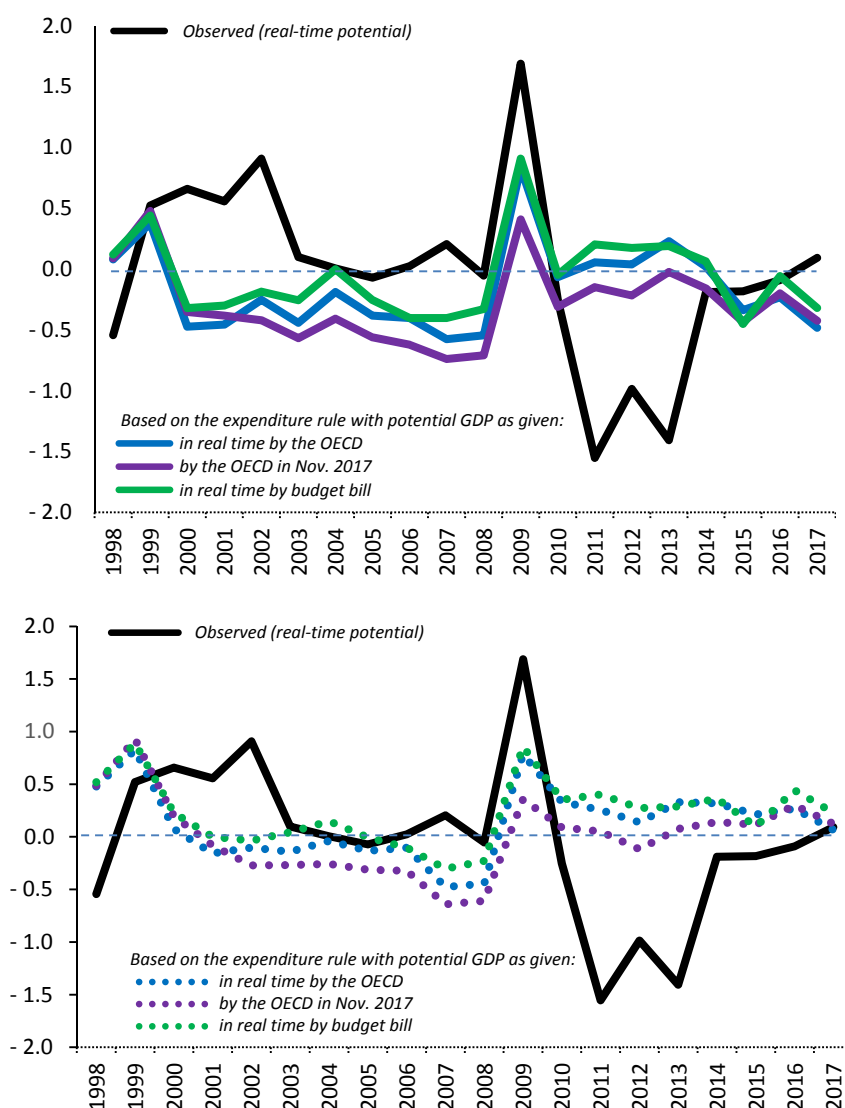
**Figure 5. Growth rate for primary public expenditure observed and predicted by the rule, in %, in current euros**



Sources: INSEE, OECD, Budget bills, OFCE calculations.

Given the evolutions of total fiscal impulses –calculated from the dynamics of the expenditures<sup>(2)</sup> which vary according to the assumptions chosen to simulate the rule and new measures in fiscal rate known for the period and exogenous to the rule of expenditure– the rule would have been more restrictive over the 2000-2009 period and more expansive over the 2010-2014 period. It is less clear for the 2015-2017 period, where in three instances it would have more expansive (with a 2% inflation target) and in three cases more restrictive (with GDP deflator forecast by the Budget bill) (Figures 6). It should be noted that the inflation target of 2% reinforces the counter-cyclical nature of the rule, which is particularly efficient during periods of “lowflation”. The impulse gaps according to the six simulations on the 2014-2017 period are mainly due to the gap between the GDP price forecast (1% on average for the period) and the inflation target at 2%.

**Figure 6. Fiscal impulse (calculated on the basis of real-time potential GDP by the Budget bill and on observed GDP prices), in points of potential GDP**



Sources: INSEE, OECD, Budget bills, OFCE calculations.

(2) To measure fiscal impulses on the expenditure side according to the different assumptions used to simulate the rule, we based our calculations on the real-time potential growth forecast in each Budget bills and the price of GDP observed.

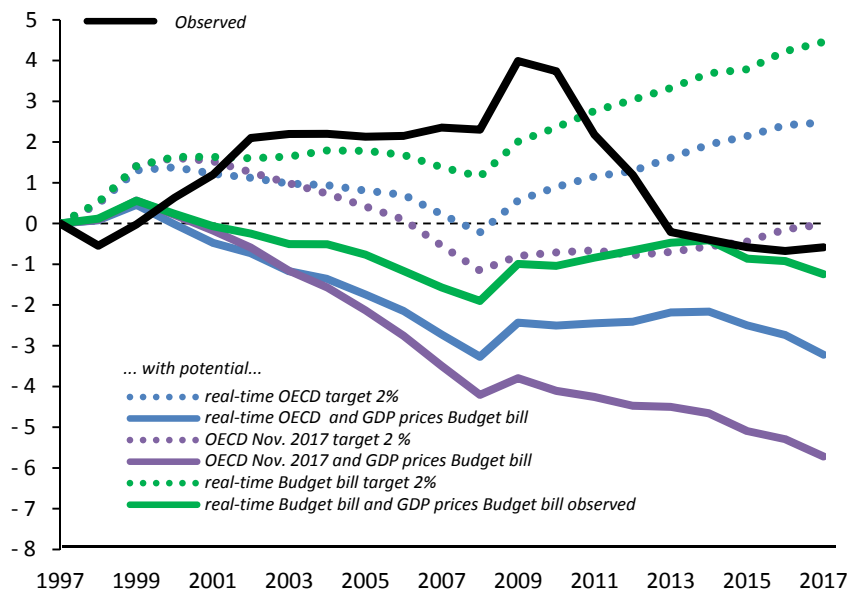
According to our calculations, based on the real-time potential GDP of the Budget bill, the cumulative fiscal impulse over the 1997-2017 period was almost neutral (-0.6 points of GDP), which is close to what our rule would give in two cases (real-time potential and deflator from the Budget bill, and potential OECD from the last vintage and 2% inflation target) (Figure 7). In two cases, the cumulative fiscal impulse would be more expansive (between 2,5 and 4,5 pts of GDP over the period) and two cases more restrictive (between - 3.2 and - 5.7 pts of GDP). According to the assumptions of potential GDP and prices, the different dynamics of the expenditure resulting from the rule lead to an impulse gap of up to 10 points of GDP in the space of twenty years.

While in some cases the accumulated budgetary impulses from the rule are close to what has been observed, the annual changes are very different and marked by two sub-periods. Indeed, over the 1997-2009 period, the cumulative fiscal impulse for France was 4 GDP points. According to the different assumptions, the spending rule would lead to more restrictive policy, ranging between - 2.0 and - 7.8 GDP points in 12 years in comparison with what it observed (Figure 7). On the contrary, the cumulative fiscal impulse was very restrictive between 2009 and 2017, at - 4.6 GDP points. In all cases, the rule would have led to a more positive fiscal impulse over this second period, ranging between + 3.5 points of GDP and + 7 points of GDP of more than observed.

However, it is important to note that the only error in the price forecasting in relation to its realization would have led to a positive fiscal impulse of + 1.9 percentage points of GDP between 2009 and 2017. And with the assumption of a 2% inflation target in the expenditure rule, the very low inflation over the period 2010-2017 would have led to a fiscal impulse of + 4.6 points of GDP. The rule can therefore lead to an expansionary fiscal policy only if the expected inflation is significantly higher than that achieved.

The differences in terms of fiscal policy would have been very important compared to what was done in the past, leading to a more negative impulse between 1997 and 2009, with consequences more positive on public finances but with a less dynamic GDP trajectory and a higher unemployment rate. After 2009, the room for fiscal measures should have been fully used as to ensure the effectiveness of the rule.

**Figure 7. Cumulative fiscal impulse over the 1997-2017 period (calculated on the basis of real-time potential GDP from the Budget bills and observed GDP), in potential GDP points**



Sources: INSEE, OECD, Budget bill, OFCE calculations.

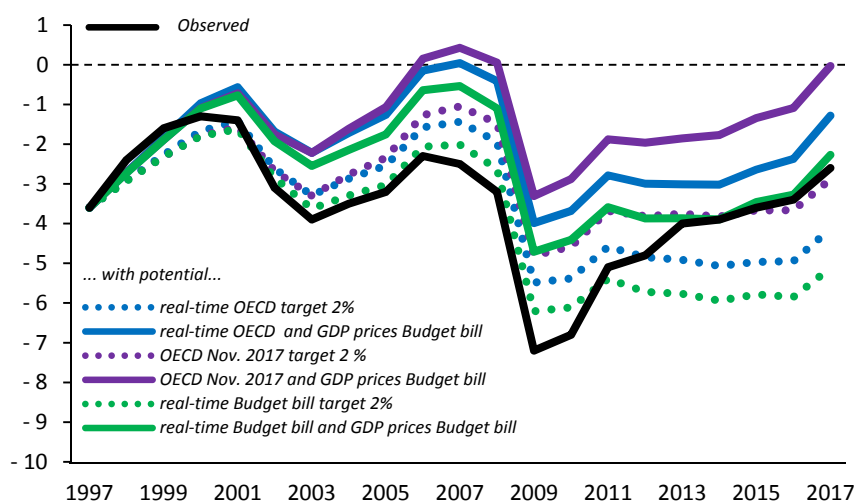
### 1.2.2. Impact on public deficit and debt

Under this rule, the trajectory of the public deficit and the public debt should have been very different. In order to simulate this trajectory, we have endogenized (which was not the case in the previous calculations) the deficit and the public debt in the rule<sup>(3)</sup> assuming a constant fiscal multiplier of 1, the initial situation being 1997<sup>(4)</sup> (Figures 8 and 9). We have not studied here the macroeconomic impacts other than those on public finances related to the application of the rule. However, these macroeconomic impacts are absolutely necessary in the understanding of the rule as a whole and the trade-offs that can arise between growth and fiscal adjustments.

The results of the simulations show that the application of the expenditure rule would, by its more counter-cyclical nature, lead to a public balance in 2007 close to balance or in surplus (against – 2.5 percentage points of GDP) in the case where the rule is simulated on the basis of price expectations from budget bills. On the other hand, on the basis of the rule, the public balance would have deteriorated significantly over the 2008-2017 period. In only two cases (out of six), it would be significantly lower in 2017 than the deficit observed and in two others significantly higher (Figure 8). Finally, in five cases simulated the six, the public debt in 2017 would be lower than the one observed due to the differences in dynamics of budget adjustments (frontloaded in the case of the rule and backloaded in observed) (Figure 9).

If the rule seems at first sight to be appealing because of its counter-cyclical nature, it must however be interpreted with great caution. On the one hand, according to the selected assumptions, it can lead to very restrictive economic choices (low potential, inflation forecast under 2%) that are harmful for growth and employment. On the other hand, over the 1997-2017 period, the restoring force has little effect on the public spending trajectory resulting from the rule, which would be quite different from a simulation of the rule starting now (from 2018), which would combine both a relatively low deficit and a high debt. This restoring force plays a fundamental long-term role on the public finances trajectory, which is developed at length in the next part.

**Figure 8. Observed budget balance and budget balance predicted by the rule over the 1997-2017 period, in GDP points**



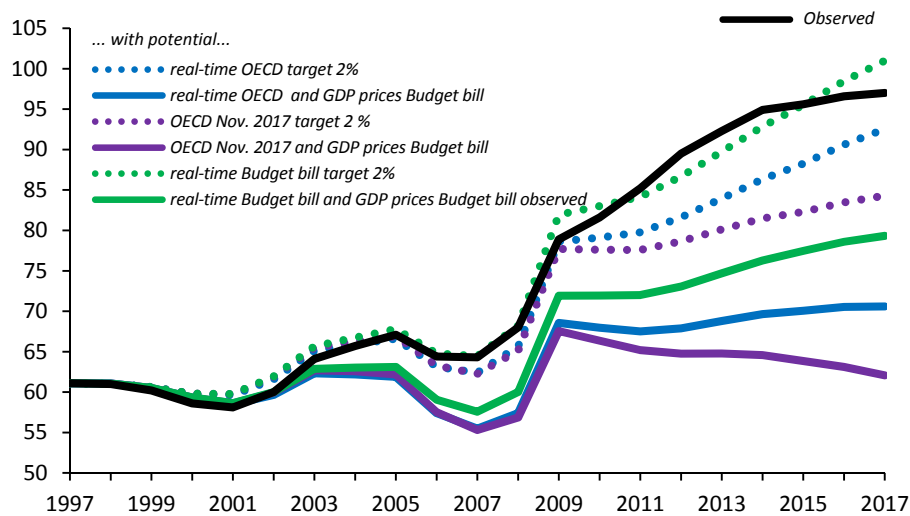
Sources: INSEE, OFCE calculations.

(3) In our simulations, the interest payments remain exogenous.

(4) Taking into account variable budget multipliers in the cycle, such as those estimated by Creel, Heyer and Plane (2011), would improve the accuracy of the simulations, but this would have a limited impact during the pre-crisis period with estimated output gaps relatively weak.



**Figure 9. Observed public debt and public debt predicted by the rule over the 1997-2017 period, in GDP points**



Sources: INSEE, OFCE calculations.

## 2. Properties of the fiscal rule

In this part, we study the properties of a fiscal rule which links the growth rate of public expenditure to the difference between the public debt level (in % of GDP) and the 60 percent of GDP debt target. Nominal public expenditure increases in pace with potential GDP plus expected inflation and a correction parameter depending on the public debt level. The rule is implemented in the iAGS model of the OFCE, which simulates the trajectory of 11 main economies of the euro area (in terms of GDP weight in the euro area).

Simulations indicate that a simple rule could not be applied to all member countries in a uniform way and reveals important flaws:

- One should take into account specific situation of each member country. The debt correction parameter could not be applied to countries with debt levels very far from the 60 percent target, such as Italy or Portugal. This would imply unrealistic fiscal efforts.
- Adjustment in public spending is activated until 60 percent debt target is realized, even if public finances are on a favorable path. It implies too much adjustment.
- The rule is asymmetric. It is silent with regard to public expenditure when debt level is under the 60 percent target. In that sense, it is not stabilizing in the long run.

In case of an unexpected shock, the rule shows different stabilizing properties according to the shock type.

- In case of an unexpected positive demand shock, the rule implies a fiscal impulse more negative than the baseline case, hence putting a brake on economic activity (and conversely in case of a negative demand shock).
- In case of an inflation shock, the rule implies a fiscal impulse more negative than the baseline case, curbing economic activity (and conversely in case of a deflationary shock).

These properties are close to the ones deriving from an application of the MTO rule, stabilizing property (or not) due to wrong anticipation of inflation, which passes on fiscal impulse (expected inflation lower than realized inflation implies a fiscal impulse lower than the one that was planned).

## 2.1. Definition of the fiscal rule

The fiscal rule deals with the growth rate of nominal public expenditure. It is written as follows:

$$\frac{G_{N,t} - G_{N,t-1}}{G_{N,t-1}} = \frac{\bar{Y}_t - \bar{Y}_{t-1}}{\bar{Y}_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}} - \gamma \left( \frac{D_{t-1}}{Y_{t-1}} - 0,6 \right) \Leftrightarrow \dot{g}_{N,t} = \dot{p}_t + \dot{\bar{y}}_t - \gamma(d_{t-1} - 0,6)$$

$G_{N,t}$ : nominal public expenditure

$\bar{Y}_t$ : potential real GDP

$P_t$ : GDP deflator

$D_t$ : public debt in euros at current prices ( $d_t$ : debt in % of GDP)<sup>2</sup>

The parameter  $\gamma$  related to public expenditure adjustment as a function to the debt gap from the target is set as follows:

$$\gamma = 0,02 \text{ if } \frac{D_{t-1}}{Y_{t-1}} > 0,9; \gamma = 0,01 \text{ if } 0,6 < \frac{D_{t-1}}{Y_{t-1}} < 0,9; \gamma = 0 \text{ if } \frac{D_{t-1}}{Y_{t-1}} < 0,6$$

We note that the rule is silent with regard to public expenditure growth when public debt is under 60 percent of GDP.

## 2.2. Implementation of the rule in iAGS

In the iAGS model, the modeling of fiscal policy is directly under the form of fiscal impulses, through expenditure and revenue measures, as a percentage of potential GDP:

$$IB_G = \frac{G_t}{\bar{Y}_t} - \frac{G_{t-1}}{\bar{Y}_{t-1}} \approx \frac{\bar{G}}{\bar{Y}} (\dot{g}_{N,t} - \dot{p}_t - \dot{\bar{y}}_t)$$

Where  $\frac{\bar{G}}{\bar{Y}}$  represents the share of public expenditure in GDP.

The fiscal rule is therefore:

$$IB_G = -\gamma \cdot \frac{\bar{G}}{\bar{Y}} \cdot (d_{t-1} - 0,6)$$

It implies that any shock affecting current prices is perfectly anticipated. As such, it therefore could not play a potential counter-cyclical role in the short run. To take into account possible unexpected shocks that would affect inflation, we separate  $\dot{p}_t$  into an expected component ( $E_{t-1}\dot{p}_t$ ) and an unexpected component  $\varepsilon_t^\pi$  calculated using current inflation due to unexpected shocks. The rule is now written as follows:

$$\dot{g}_{N,t} = E_{t-1}\dot{p}_t + \dot{\bar{y}}_t - \gamma(d_{t-1} - 0,6)$$

Hence

$$IB_G = \frac{\bar{G}}{\bar{Y}} \cdot [E_{t-1}\dot{p}_t - \dot{p}_t - \gamma \cdot (d_{t-1} - 0,6)] = \frac{\bar{G}}{\bar{Y}} \cdot [-\varepsilon_t^\pi - \gamma \cdot (d_{t-1} - 0,6)]$$

In case of inflation surprise, fiscal impulse on expenditure is more negative, because expected prices without shock implies a lower growth rate of public expenditure than the *ex post* nominal growth rate of potential GDP.

## 2.3. Definition of the baseline case

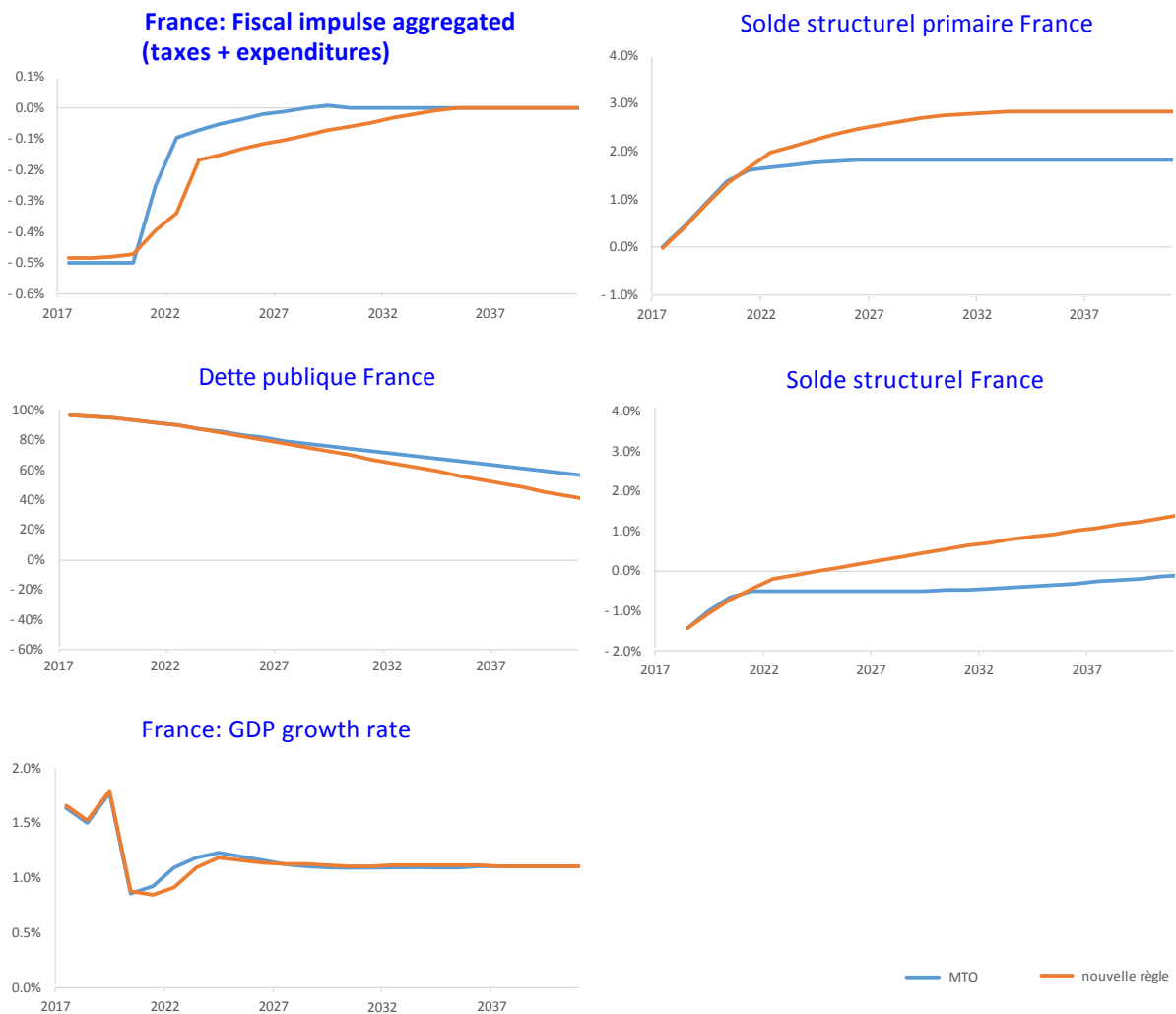
The baseline case integrates simulations which have been done for the 2018 iAGS Report (2018-2019 Economic Outlook). Fiscal impulses are calculated to comply with the MTO rule. They are divided into two parts: 50% in expenditure ( $IB_{G,MTO}$ ) and 50% in revenue ( $IB_{T,MTO}$ ).

### Variant 1.1: The rule is implemented in France only

In this variant, the fiscal rule related to nominal expenditure growth rate replaces the trajectory of fiscal impulse in expenditure as from 2017 in France. Fiscal impulse in revenue are kept and neutralized in the expenditure rule so that only spending multipliers are at play:

$$IB_{G,FRA} = -\gamma \cdot \frac{\bar{G}_{FRA}}{\bar{Y}_{FRA}} \cdot (d_{t-1,FRA} - 0,6) - IB_{T,MTO,FRA}$$

Figure variant 1.1

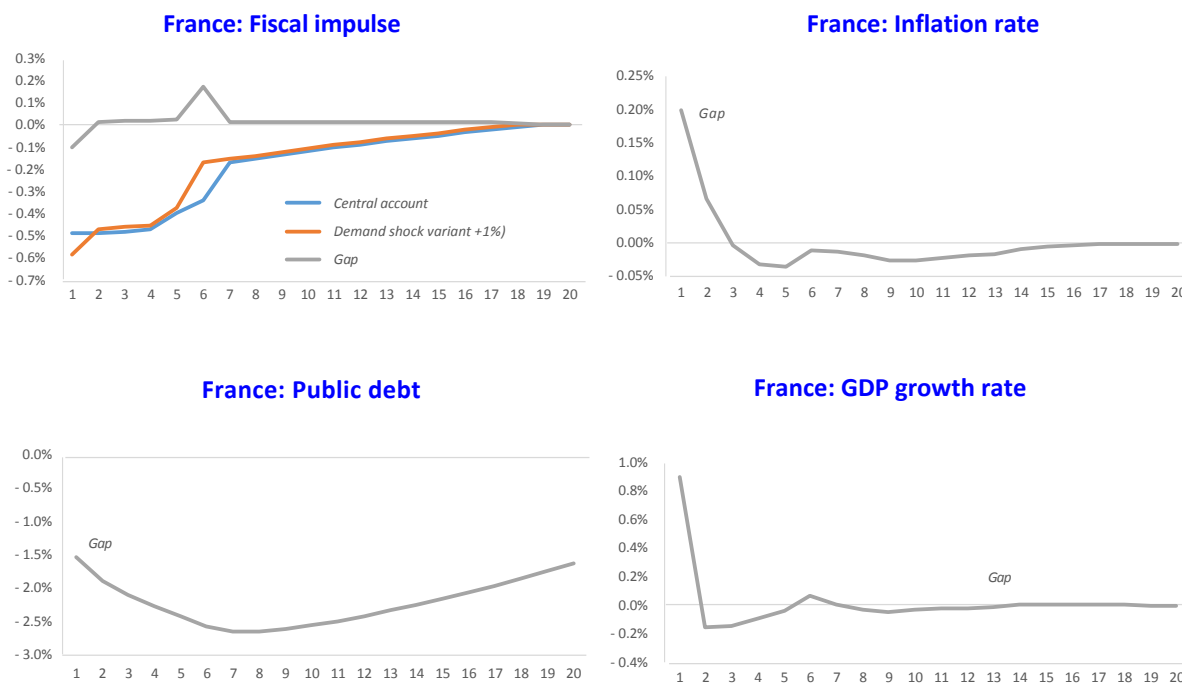


Source: iAGS model, OFCE.

### Variant 1.2: The rule is implemented in France only - + 1% demand shock

In this variant, we simulate a positive unexpected 1% shock on aggregate demand. The shock is temporary (one period). This variant is a way to assess the property of the rule in case of a temporary unexpected demand shock.

Figure variant 1.2

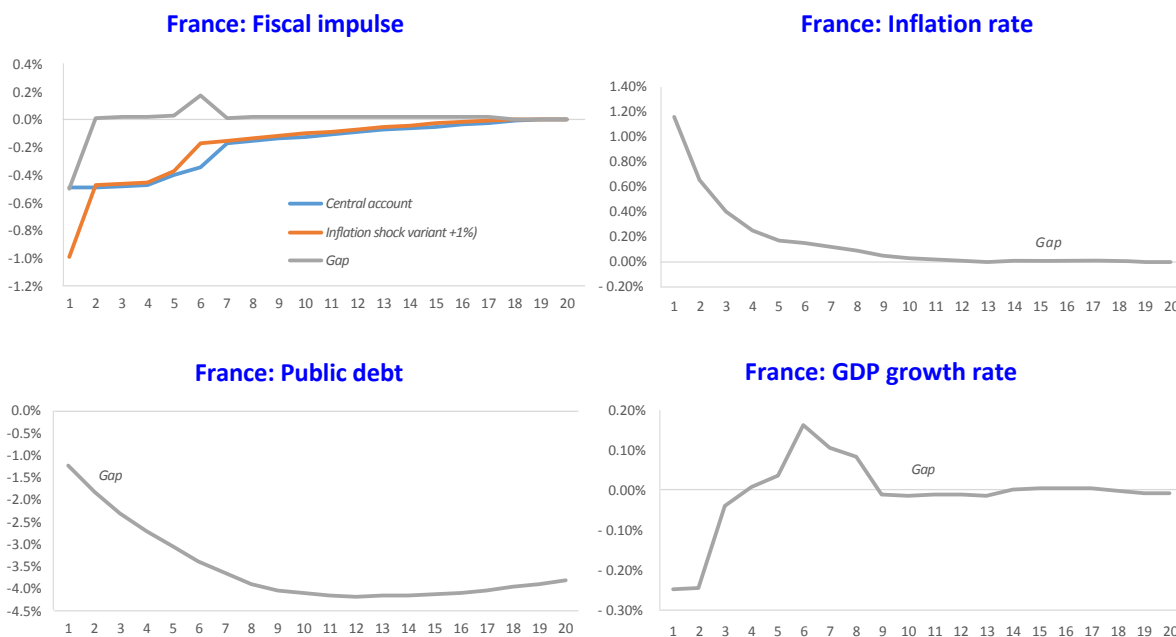


Source: iAGS model, OFCE.

### Variant 1.3: The rule is implemented in France only - + 1% inflation shock

In this variant, we simulate a positive unexpected + 1% shock on inflation. The shock is temporary (one period). This variant is a way to assess the property of the rule in case of a temporary unexpected inflation shock.

Figure variant 1.3

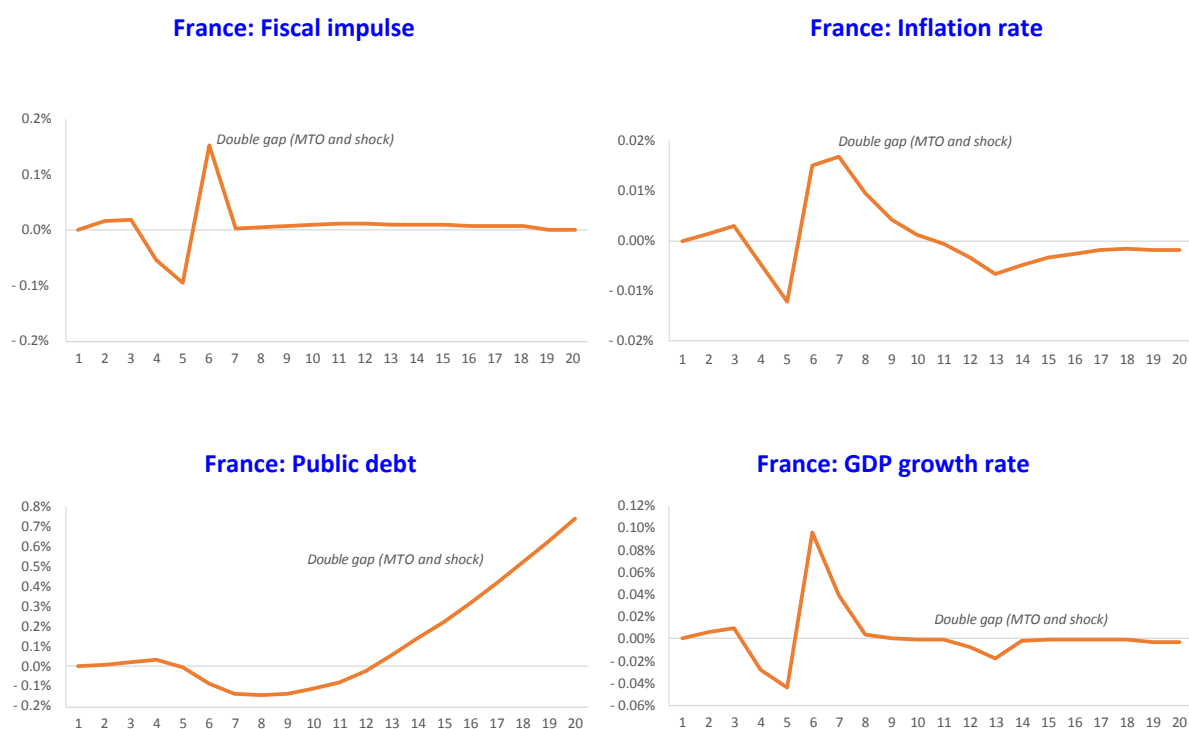


Source: iAGS model, OFCE.

### Variant 1.4: Comparison MTO /expenditure rule following a demand shock of + 1%

In this variant, we simulate a positive unexpected shock of 1% on aggregate demand, and application of the MTO rule or the expenditure rule. The shock is temporary (one period). The global effect of the simulation comprises one effect due to the differences in fiscal rule (MTO versus expenditure rule – variant 1) and one effect due to the different reaction to the shock. The effect we are looking for is the one related to relative capacity of the expenditure rule to stabilize GDP and inflation, therefore we subtract from the global effect the effect due the differences in applied rule. This variant is therefore a way to assess the capacity of the rule to stabilize GDP and inflation after an unexpected positive demand shock.

Figure variant 1.4

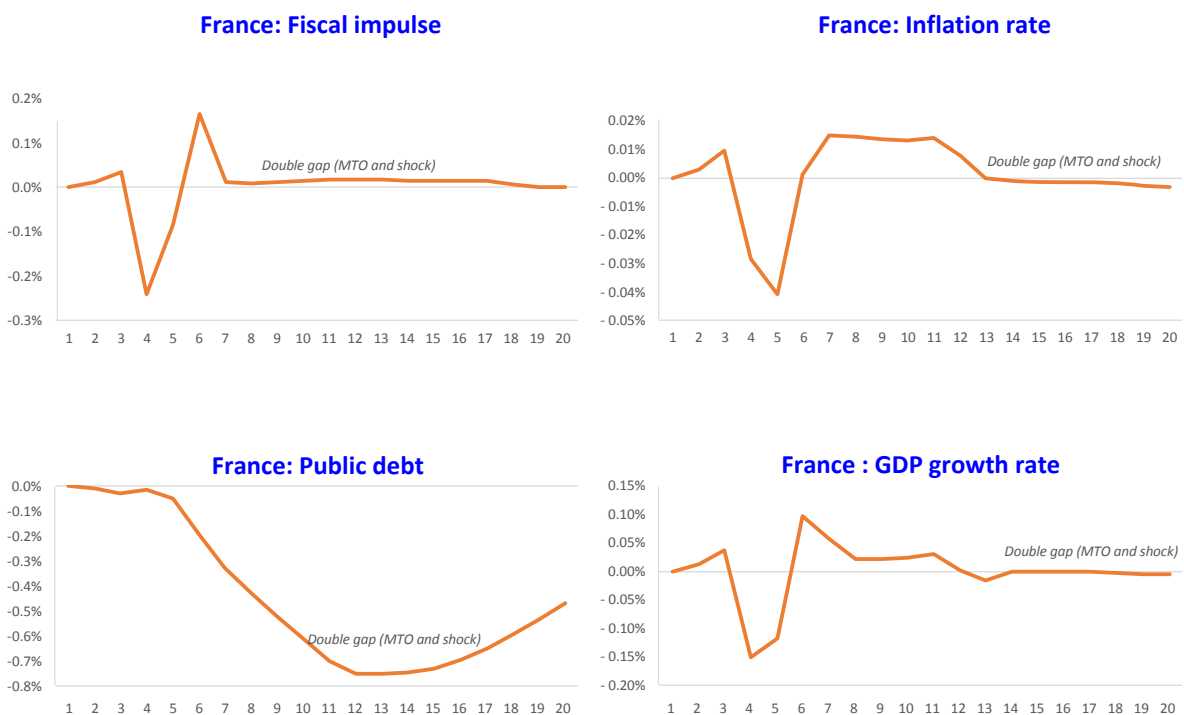


Source: iAGS model, OFCE.

### Variant 1.5: Comparison MTO / expenditure rule following an inflation shock of + 1%

In this variant, we simulate a positive unexpected 1% shock on inflation, and application of the MTO rule or the expenditure rule. The shock is temporary (one period). The global effect of the simulation comprises one effect due to the differences in fiscal rule (MTO versus expenditure rule – variant 1) and one effect due to the different reaction to the shock. The effect we are looking for is the one related to relative capacity of the expenditure rule to stabilize GDP and inflation, therefore we subtract from the global effect the effect due the differences in applied rule. This variant is therefore a way to assess the capacity of the rule to stabilize GDP and inflation after an unexpected positive inflation shock.

Figure variant 1.5



Source: iAGS model, OFCE.

### **Variant 1.6: The rule is implemented in all countries of the euro area**

In this variant, the fiscal rule related to nominal expenditure growth rate replaces the trajectory of fiscal impulse in expenditure as from 2017 in all countries (France, Germany, Italy, Spain, Austria, Belgium, Ireland, Netherland, Greece, Portugal, Finland). Fiscal impulse in revenue are kept and neutralized in the expenditure rule so that only spending multipliers are at play.

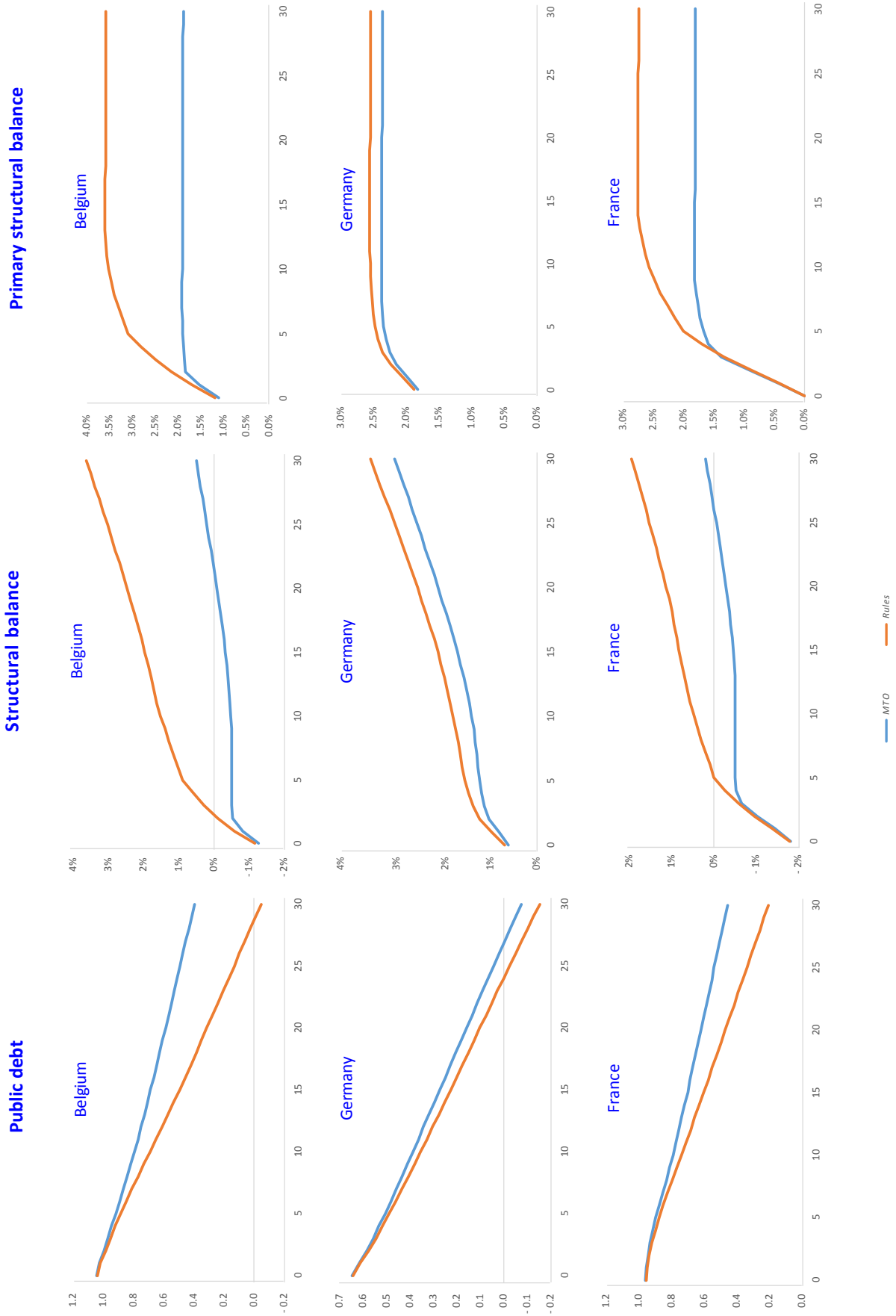
In the first set of Figures 6.1, the parameter related to public expenditure adjustment as a function to the debt gap from the target is  $\gamma = 0,02$  if  $\frac{D_{t-1}}{Y_{t-1}} > 0,9$ ;  $\gamma = 0,01$  if  $0,6 < \frac{D_{t-1}}{Y_{t-1}} < 0,9$ ;  $\gamma = 0$  if  $\frac{D_{t-1}}{Y_{t-1}} < 0,6$ . The rule implies heavy adjustments for countries such as Italy or Portugal in terms of primary cyclically-adjusted balance: close do 10% of GDP in Italy and around 9% of GDP in Portugal.

In the second set of Figures 6.2, the parameter related to public expenditure adjustment as a function to the debt gap from the target is  $\gamma = 0,001$  if  $\frac{D_{t-1}}{Y_{t-1}} > 0,6$  and  $\gamma = 0$  if  $\frac{D_{t-1}}{Y_{t-1}} < 0,6$ . The rule implies a more realistic adjustment for Italy and Portugal, but very slow for France.

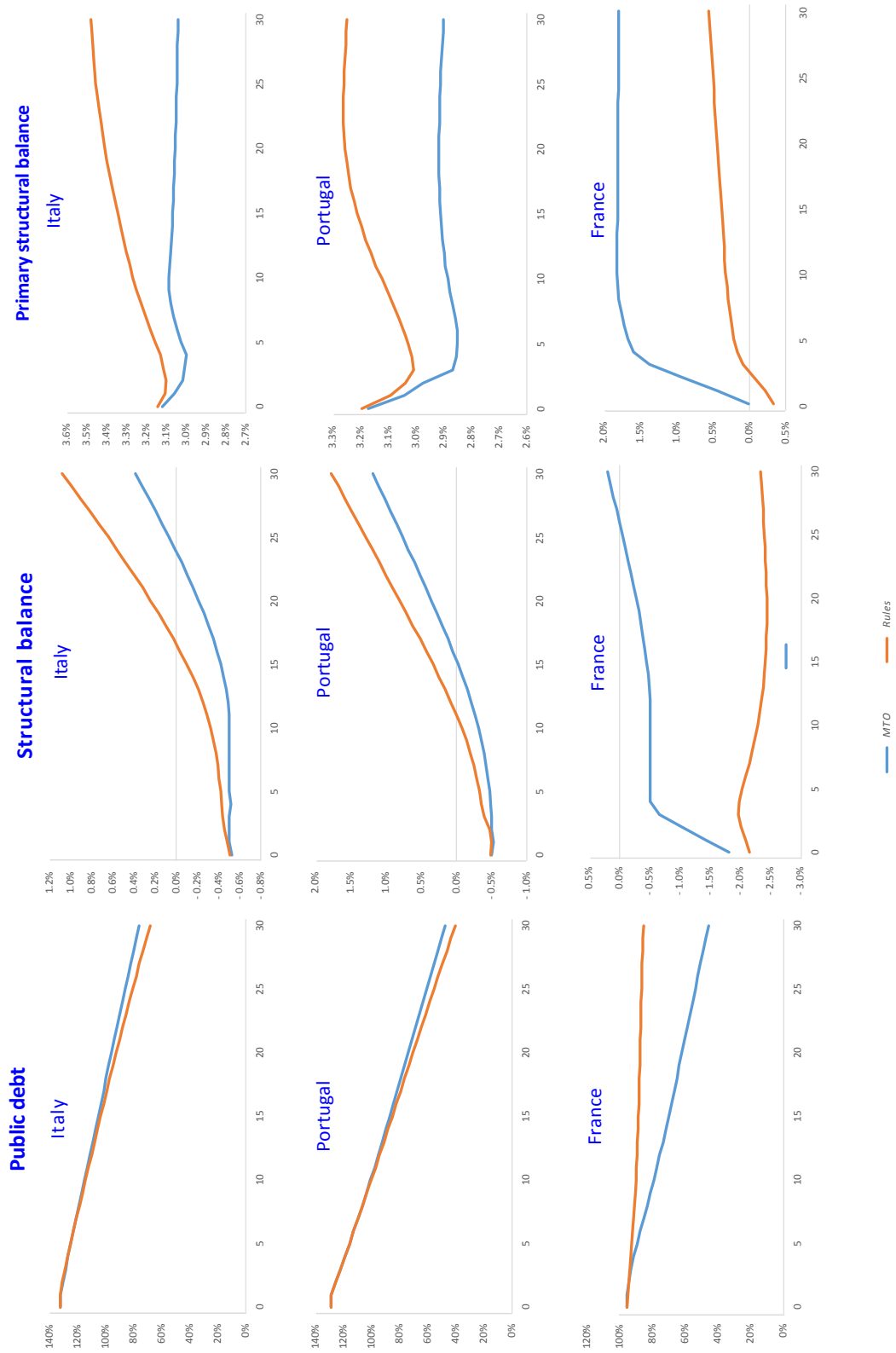
These results indicate that such a rule should be country specific. In any case, it remains asymmetric and implies an adjustment which seems too important: after a few years, as the debt is curbing, the rule implies to continue fiscal efforts, which does not make economic sense.



Figure variant 1.6.1:  $\gamma = 0,02$  if  $\frac{D_{t-1}}{Y_{t-1}} > 0,9$ ;  $\gamma = 0,01$  if  $0,6 < \frac{D_{t-1}}{Y_{t-1}} < 0,9$ ;  $\gamma = 0$  if  $\frac{D_{t-1}}{Y_{t-1}} < 0,6$



**Figure variant 1.6.2:**  $\gamma = 0,001$  if  $\frac{D_{t-1}}{Y_{t-1}} > 0,6$ ;  $\gamma = 0$  if  $\frac{D_{t-1}}{Y_{t-1}} < 0,6$



Source: iAGS model, OFCE.

### 3. Debt reduction objective at a given time horizon and fiscal effort

Starting from a debt reduction objective (in % of GDP) for a given time horizon, we compute the corresponding fiscal effort every year. The objective can either be understood as the absolute variation with respect to a given starting point or as the absolute variation with respect to the projected public debt trajectory.

In the first part, we show that this effort depends crucially on various aspects:

- The time-horizon. The closer this time-horizon, the tighter the constraint, the tougher the effort for a given objective;
- The initial situation of the country when the rule is applied. More precisely, the effort is bigger for large primary deficit and for big gaps between the interest rate and the growth rate;
- The objective. The more ambitious the objective (far from the initial level of public debt) the tougher the effort can be. Yet, this also depends on the public debt trajectory net of fiscal adjustment that depends on the previous point.

In the second part, we illustrate the application of such a rule on several countries of the euro zone with heterogeneous initial characteristics. These simulations are based on the OFCE's iAGS model.

#### 3.1. Analysis of the public debt dynamic

##### 3.1.1. Public Debt dynamic and fiscal rule

The debt dynamic comes from the following equation (capital letters denote nominal variables, S is the primary balance, D the financial public debt):

$$D_t = (1 + r_t)D_{t-1} - S_t \#(1)$$

Dividing by the value of potential production ( $\tilde{Q}_t$ ) and by defining the potential growth rate as  $\tilde{Q}_{t-1}/\tilde{Q}_t = \frac{1}{1+g_t}$ , we get:

$$\Delta d_t = \frac{r_t - g_t}{(1 + g_t)} d_{t-1} - s_t \#(2)$$

The primary balance that stabilizes the debt at  $d^*$  can thus be deduced as:

$$s^* = \frac{r - g}{(1 + g)} d^* \#(3)$$

As long as  $r > g$ , this balance is positive and increasing with the public debt. On the contrary, if  $g < r$ , the balance is negative (budget deficit) This is even more the case, when the debt is high. This condition can be expressed in terms of effective balance (including interest payments).

$$s_e^* = s^* - r d^* = -\frac{g + r g}{(1 + g)} d^* \cong -g d^* \#(4)$$

This expression justifies a public deficit of 3% of GDP for a debt of 60% and a nominal growth rate of 5%. By defining  $\lambda = \frac{r-g}{(1+g)}$ ,  $\hat{s}_t = s_t - s^*$  and  $\hat{d}_t = d_t - d^*$  joining equation (2) and (4) yields:

$$\Delta \hat{d}_t = \lambda_t \hat{d}_{t-1} - \hat{s}_t \#(5)$$

If  $\lambda$  is time-independent:

$$\hat{d}_{t+T} = (1 + \lambda)^T \hat{d}_{t-1} - \sum_{i=0, T} (1 + \lambda)^{T-i} \hat{s}_{t+i} \quad \#(6)$$

If  $\lambda = 0$ , we have a simple expression:

$$\hat{d}_{t+T} = \hat{d}_{t-1} - \sum_{i=0, T} \hat{s}_{t+i}$$

We now have to determine the fiscal impulse needed to reduce the debt by a given quantity. Starting from  $t=1$ , we look for  $ds$  such that  $\hat{d}_T - \hat{d}_0 = -\delta$  and such that  $\hat{s}_t = \hat{s}_{t-1} + ds$  ( $0 < t \leq T$ ). This yields:

$$ds = \frac{\delta + ((1 + \lambda)^T - 1)d_0 - \sum (1 + \lambda)^{T-i} \hat{s}_0}{\sum i(1 + \lambda)^{T-i}} \quad \#(7)$$

Quite logically,  $ds$  is increasing in the target, depends on the initial level of debt depending on the snowball effect as well as of the initial primary balance. The numerator being equal to  $T^2$ , the more distant the time-horizon, the smaller the  $ds$ .

When  $\lambda = 0$  the expression is simpler:

$$ds = \frac{2}{T + 1} \left( \frac{\delta}{T} - \hat{s}_0 \right) \quad \#(8)$$

Note that this expression does not depend on  $d_0$  but depends on  $\hat{s}_0$ . By iterating the objective (reduction of  $\delta$  of the debt ratio at time 1 followed by a similar reduction at time 2 and so on...), we get:

$$\begin{aligned} ds_i &= \frac{2}{T + 1} \left( \frac{\delta}{T} - \hat{s}_{i-1} \right) \\ &= \frac{2}{T + 1} \left( \frac{\delta}{T} - (\hat{s}_{i-2} - ds_{i-1}) \right) \\ &= \frac{T - 1}{T + 1} ds_{i-1} \quad \#(9) \end{aligned}$$

When  $\lambda$  is different from 0, this recurrence formula is given by:

$$ds_i = \left[ 1 - \frac{\sum (1 + \lambda)^{T-i}}{\sum i(1 + \lambda)^{T-i}} \right] ds_{i-1} \quad \#(10)$$

It can be deduced that the sequence of  $ds_i$  is decreasing.

The interpretation goes as follow: the debt reduction rule converges at infinity toward a 0  $ds_i$ . It corresponds to the constant value of the balance that ensures a constant reduction of the debt. If  $\delta$  is null, this balance is necessarily the debt-stabilizing balance. As expected, this type of rule converges and yields a relatively smooth sequence of fiscal impulses. The rule tends to frontload the impulses as the  $ds_i$  are decreasing. The bigger effort is realized at the beginning of the time frame. This is even more so when  $\lambda$  is negative or when  $T$  is small.

$$z(T, \lambda) = 1 - \frac{\sum (1 + \lambda)^{T-i}}{\sum i(1 + \lambda)^{T-i}} < 1, \text{ tends toward 1 when } T \rightarrow +\infty, \text{ is increasing in } T \text{ and in } \lambda.$$

### 3.1.2. An application

We offer a first application that directly relates to the previous analysis, for four countries of the euro zone (France, Germany, Italy and Spain). The equation (7) allows us to approximate the fiscal efforts to reach the required debt reduction. A tax gap can be defined as the constant effort,  $ds_i = ds$ , to be done each year during  $T$  years in order to reach either a given debt reduction (denoted  $\hat{\delta} = -(d_T(ds_i = ds) - d_T(ds_i = 0))$ ) with respect to the situation where no additional effort are done ( $ds_i = 0$ ), or for a debt reduction with respect to its initial level, denoted  $\delta = -(d_T(ds_i = ds) - d_0)$  (that is a positive  $\delta$ ). Let  $\hat{t}g$  be the first tax gap and  $tg$  the second one.

Formally, it can be deduced from equation (6):

$$d_T(ds) = \tau(T, \lambda)d_0 - \theta(T, \lambda)s_0 - \Theta(T, \lambda)ds \quad (11)$$

By denoting:  $(T, \lambda) = (1 + \lambda)^T$ ,  $\theta(T, \lambda) = \sum_{i=1, T} (1 + \lambda)^{T-i}$ ,  $\Theta(T, \lambda) = \sum_{i=1, T} i(1 + \lambda)^{T-i}$

We get:

$$\hat{t}g(T, \delta) = \frac{\delta}{\Theta} \text{ et } tg(T, \delta) = \frac{\delta - (1 - \tau)d_0 - \theta s_0}{\Theta} \quad (12)$$

### An illustration of this model is given by the following numerical application

	$T = 5$				$T = 10$			
	FRA	ITA	ESP	DEU	FRA	ITA	ESP	DEU
$\hat{t}g(T, 5)$	0.51	0.48	0.49	0.52	0.12	0.10	0.11	0.12
$tg(T, 5)$	0.31	0.15	0.25	-0.55	0.03	-0.04	0.00	-0.36
$\hat{t}g(T, 10)$	1.02	0.95	0.98	1.04	0.23	0.20	0.21	0.24
$tg(T, 10)$	0.82	0.63	0.74	-0.03	0.14	0.06	0.11	-0.24

### OECD data used for the calibration, 2016

	FRA	ITA	ESP	DEU
Output gap of the total economy	-2.1	-3.2	-5.2	1.1
Long-term interest rate on government bonds	0.5	1.5	1.4	0.1
Potential GDP growth (volume)	1.2	-0.1	0.6	1.3
Government primary balance, as a percentage of GDP	-1.7	1.3	-2.0	1.8
Cyclically adjusted government primary balance, as a percentage of potential GDP	-0.3	2.8	1.3	1.3
Gross public debt, Maastricht criterion, as a percentage of GDP	96.6	131.9	99.0	68.2

Note that the results of the previous simulations do not depend on the first output gap. In other words, when Spain closes its output gap, it also requires improving the future conjuncture fiscal balance. In addition, the fiscal policy creates a feedback loop on the short-term growth rate through the fiscal multiplier, which in turn distorts in the short-term the debt ratio expressed in ratio of current GDP (and not *potential GDP*). These technicalities are taken into account in the following simulations.

## 3.2. Debt Dynamics and fiscal rule: an application based on the iAGS model

### 3.2.1. Definition of the fiscal policy objectives

The aim of fiscal policies is to make public debt fluctuating at a given magnitude of  $\Delta$  point of GDP at a given time horizon  $h$ . The fiscal policy is decided at each period ( $h$  does not vary at each period).

- At  $t$ , an average fiscal impulse exclusively oriented on public spending is decided for the period  $t$  to  $t+h-1$ . Therefore, the projection of this public debt variation between  $t-1$  and  $t+h$  is equal to  $\Delta$ .
- At  $t+1$ , this previous step is repeated in order to calculate a new sequence of public expenditures compatible with a  $\Delta$  variation of public debt between  $t$  and  $t+1$ .
- And so on...

As previously shown, the implementation of this kind of fiscal policies is highly conditioned by the initial trajectory of public debt (the effort implied in order to reduce public debt by  $\Delta$  points is not the same depending on trend of the trajectory on which the debt is projected (stable, increasing or decreasing). Put differently, the plausible fiscal effort is conditioned by the starting point, that is, by the distance between the initial fiscal balance and the debt-stabilizing fiscal balance. It is also conditioned by the gap between the interest rate and the GDP growth rate (snowball effect) and by the time horizon,  $h$  (for very distant time horizon, the effort can be spread  $t$  and  $t+h-1$ ).

The computation of a new public spending trajectory at each period of time does not ensure that the past variation of public debt is on average consistent with the debt variation target between  $t-1$  and  $t+h$ . A sequence of negative fiscal impulses constitutes a burden on growth and on public receipt in the short-run through the fiscal multiplier and the effect of automatic stabilizers, which slows down the initial decrease of public debt.

In the iAGS model, the fiscal policy is directly modeled as fiscal impulses in spending and in compulsory levy, as percentage of potential GDP.

$$FI_G = \frac{G_t}{\bar{Y}_t} - \frac{G_{t-1}}{\bar{Y}_{t-1}} \approx \frac{\bar{G}}{\bar{Y}} (\dot{g}_{N,t} - \dot{p}_t - \dot{\bar{y}}_t)$$

$FI_G$ : Fiscal impulse on public spending

$G_{N,t}$ : Public spending in values

$\bar{Y}_t$ : Potential GDP in volume

$P_t$ : GDP price

$\frac{\bar{G}}{\bar{Y}}$ : Share of public spending in the GDP

One percentage point of negative fiscal impulse corresponds to a one-percentage point decrease of the public spendings/potential GDP ratio. The condition to get the fiscal impulse at each period  $j$  writes:

$$FI_{G;j:j+h-1} \quad tq \quad \frac{D_{j+h}}{Y_{j+h}} - \frac{D_{j-1}}{Y_{j-1}} = \Delta$$

$D_t$ : Public debt expressed in current euros.

### 3.2.2. Definition of central account

The central account is based on the firm-level simulation realized in the 2018 iAGS report (growth forecast for 2018-2019). The fiscal impulses are then calculated in order to respect the MTO. These fiscal impulses are divided: 50% on spending ( $FI_{G,MTO}$ ) and 50% on compulsory levy ( $FI_{T,MTO}$ ).

#### 1<sup>st</sup> simulation: application of the rule for France only

In this simulation, the sequence of fiscal impulses calculated at each period substitutes itself to the trajectory of fiscal impulses in spending for France from 2017 onward. The sequence of fiscal impulses is calculated each year between 2017 and 2025. The fiscal impulses in compulsory levy (which by convention accounts for half of the effort to be done to reach the MTO in the central account) are set to 0 in projection. The reason for that is that in this scenario, we don't want the adjustment to be done by an increase of the taxes.

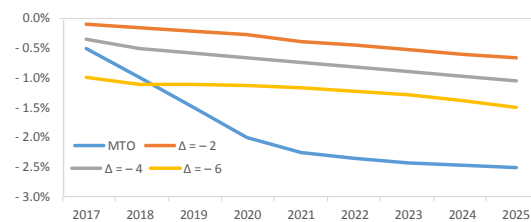
In this iAGS model, the fiscal multiplier on public spending is higher than the one on compulsory levy (accordingly with the related empiric literature). With a too negative sequence of fiscal impulses the model has no closed form solutions (this can be the case when a very negative  $\Delta$  is combined with a low  $h$  and a deteriorated structural budget balance).

As an example, we run three set of simulations based on the iAGS model by setting  $h = 5$  ans for  $\Delta = -2$ ,  $\Delta = -4$  and  $\Delta = -6$ .

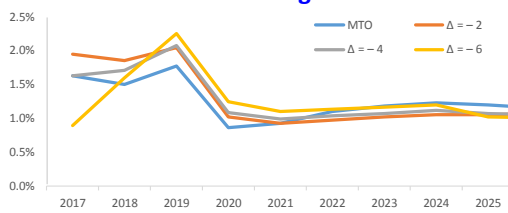
We also restrict the negative fiscal impulse on public spending at a level of  $-1$  the first year. In the case where  $\Delta = -6$ , the objective is not reached the first year but is easily reached afterwards.

**Figure Simulation 2.1**

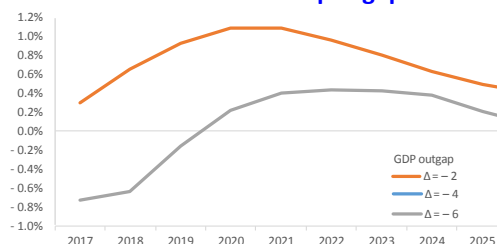
#### Evolution of public spending in % of potential GDP



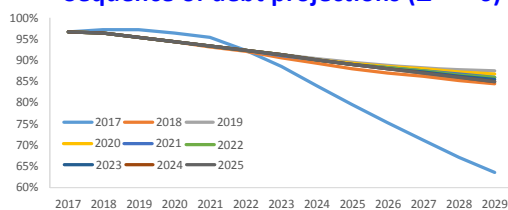
#### France: GDP growth rate



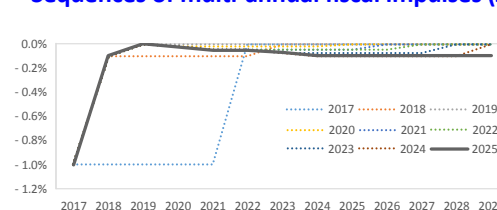
#### France: Output gap



#### Sequence of debt projections ( $\Delta = -6$ )



#### Sequences of multi-annual fiscal impulses ( $\Delta = -6$ )



Source: iAGS model, OFCE's calculations.

The calculation of the sequence of fiscal impulses by iteration every year imply that the fiscal impulse decided in the first year to more negative when the debt reduction target is ambitious. The decrease of public debt is not linear. This decrease is lower the first year than the average decrease over the period. The second year, when the trajectory is updated, public debt should be decreased again but the time horizon changes and the previous variation in debt level (that of year 1) is “forgotten”. The sequence of fiscal impulses can therefore be significantly revisited (e.g. in case of  $\Delta = -6$ ).

The trajectory of the public spending over potential GDP ratio can be deducted from the cumulative public spending (and not compulsory levy) fiscal impulses.

In order to obtain a debt trajectory closed to the MTO baseline on the medium-run,  $\Delta$  should be approximately set to  $-10$ . The implied adjustment of the short-run effort is high and  $\Delta$  is less than  $-10$  the two first years.

### 2<sup>nd</sup> simulation: application of the rule for France, Germany, Italy, Spain and Portugal

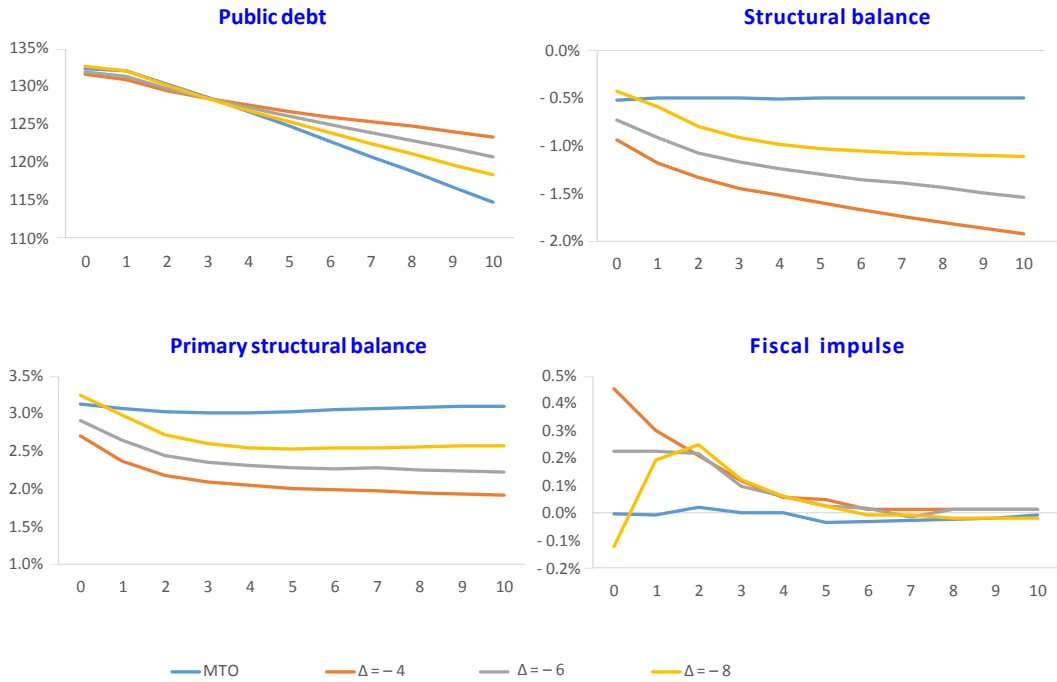
In this simulation, the sequence of fiscal impulses calculated each year replace the fiscal impulse trajectory (in public spending) for France, Germany, Italy, Spain and Portugal from 2017 onward. The sequence of fiscal impulse is computed each year between 2017 and 2025. Germany and Portugal being on the favorable debt trajectory, their  $\Delta$  is higher ( $\Delta \in \{-14; -16; -18\}$  for Germany and  $\Delta \in \{-10; -12; -14\}$  for Portugal). We report these results for Spain and Italy with  $\Delta \in \{-4; -6; -8\}$ .

In the baseline scenario (MTO), Italy, Germany and Portugal do not have any effort to do. The structural budget balance is slightly above 0,5 point of GDP and the debt decreases in projection. Introducing a debt target at a 5-years-time horizon, should therefore be read in comparison to the projected debt decrease. A sharper decrease implies efforts in spending, while a lower decrease implies more flexibility for fiscal policies. For instance, in the case of Germany, setting  $\Delta = -14$  leads to loose fiscal policy, whereas it should be restrictive for  $\Delta = -18$ .

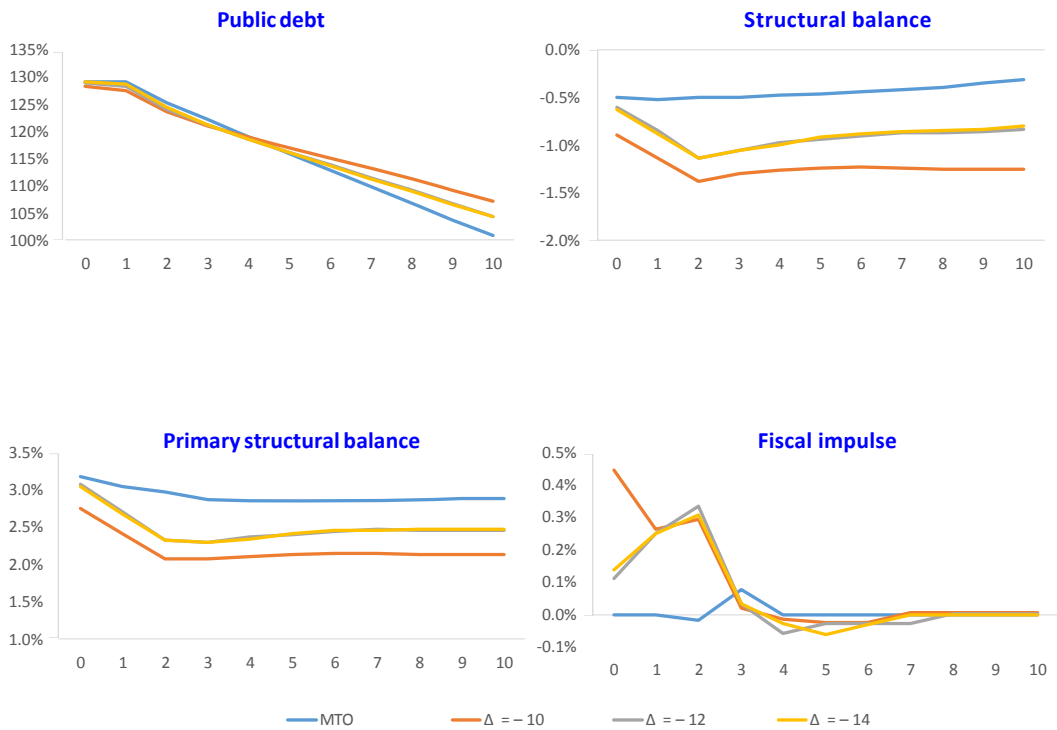


Figure simulation 2.2

Italy



Portugal



Source: iAGS model, OFCE's calculations.